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First Semester B.E. Degree Examination, Dec.2023/Jan.2024 Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the n^{th} derivative of $\frac{6x}{(x-2)(x+2)(x-1)}$. (06 Marks)
- b. Prove that the curves $r = a(1 + \cos\theta)$, $r = b(1 - \cos\theta)$ cuts orthogonally. (07 Marks)
- c. Find the radius of curvature for the curve $y^2 = \frac{4a^2(2a-x)}{x}$, where the curve cuts the x-axis. (07 Marks)

OR

- 2 a. If $y = a \cos(\log x) + b \sin(\log x)$ then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (06 Marks)
- b. In usual notation, prove that $\tan \phi = r \frac{d\theta}{dr}$. (07 Marks)
- c. Find the Pedal equation of the curve $r^n = a^n \cos n\theta$. (07 Marks)

Module-2

- 3 a. Obtain the Taylor's series of $\log_e x$ in powers of $(x-1)$ upto fourth degree. (06 Marks)
- b. State Euler's theorem, use the same prove that $xu_x + yu_y = 2u \log u$ where $\log u = \frac{x^3 + y^3}{3x + 4y}$. (07 Marks)
- c. If $u = x + 3y^2 - z^2$, $v = 4x^2 yz$, $w = 2z^2 - xy$. Evaluate $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at $(1, -1, 0)$. (07 Marks)

OR

- 4 a. Evaluate $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{\frac{1}{x}}$. (06 Marks)
- b. Find the Maclaurians expansion of $\log(\sec x)$ upto fourth degree term. (07 Marks)
- c. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$. (07 Marks)

Module-3

- 5 a. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$. Find the velocity and acceleration at $t = 1$. Also find component of velocity and component of acceleration in the direction of $\hat{i} + 3\hat{j} + 2\hat{k}$. (06 Marks)
- b. Find constants 'a' and 'b' such that $\vec{F} = (axy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (bxz^2 - y)\hat{k}$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. (07 Marks)
- c. Find the angle between the tangents to the curve $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$ at the point $t = \pm 1$. (07 Marks)

OR

- 6 a. Find $\text{div } \vec{f}$ and $\text{curl } \vec{f}$ where, $\vec{f} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. (06 Marks)
- b. If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, prove that $\nabla(r^n) = nr^{n-2}\vec{r}$. (07 Marks)
- c. Prove that $\nabla \cdot (\phi \vec{A}) = \phi (\nabla \cdot \vec{A}) + \nabla\phi \cdot \vec{A}$. (07 Marks)

Module-4

- 7 a. Obtain the reduction formula for $\int \sin^n x \, dx$, hence evaluate $\int_0^{\frac{\pi}{2}} \sin^n x \, dx$. (06 Marks)
- b. Solve $\left(1 + e^x\right) dx + e^y \left(1 - \frac{x}{y}\right) dy = 0$. (07 Marks)
- c. If the temperature of the air is 30°C and the substance cools from 100°C to 70°C in 15 minutes, find when the temperature will be 40°C . (07 Marks)

OR

- 8 a. Evaluate $\int_0^{\frac{\pi}{6}} \cos^4 3x \sin^2 6x \, dx$. (06 Marks)
- b. Solve $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$. (07 Marks)
- c. Find the orthogonal trajectory of $r^n = a^n \cos n\theta$. (07 Marks)

Module-5

- 9 a. Find the rank of the matrix,

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

Using elementary row operations. (06 Marks)

- b. Solve $2x + y + 4z = 12$, $4x + 11y - z = 33$, $8x - 3y + 2z = 20$ by Gauss elimination method. (07 Marks)

- c. Find the largest eigen value and the corresponding eigen vector for $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ taking the initial vector as $[1 \ 1 \ 1]^T$ carry out 5 iterations by using power method. (07 Marks)

OR

- 10 a. Solve $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$ by Gauss-Seidel method. Carry out 3 iterations. (06 Marks)
- b. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form. (07 Marks)
- c. Reduce the quadratic form, $3x^2 + 5y^2 + 3z^2 - 2xy + 2zx - 2yz$ to the canonical form, using orthogonal transformation. (07 Marks)
